THE COMPLETE E8 × E8 LAGRANGIAN

FROM 496 SYMMETRIES TO OUR UNIVERSE — IN ONE ACTION

THE MASTER ACTION — 10D STRING FRAME

$$\mathcal{S} = 1/2\kappa^2{}_{10} \int d^{10}x \sqrt{(-g)}\, e^{-2f} [\mathcal{R} + 4(\partial\Phi)^2 - \tfrac{1}{2}|H_3|^2 - \alpha'/4(\mathrm{tr}|F_1|^2 + \mathrm{tr}|F_2|^2)]$$
$$+ [\bar{\psi}_m \Gamma^{MNP} \mathcal{D}_n \psi_p + \bar{\lambda}\Gamma^M \mathcal{D}_m \lambda + \alpha'/4\ \bar{\chi}^a \Gamma^M \mathcal{D}_m \chi^a] + (\text{Yukawa} + 4\text{-fermi})$$

Gravity · B-field · Visible E_8 · Hidden E_8 · Fermions — 7 fields · 3 lines · entire universe

§1 THE 7 FUNDAMENTAL FIELDS

4 bosons + 3 fermions = everything

g_{MN} — 10D Metric
Graviton · 35 DOF
→ 4D graviton + moduli scalars

B_{MN} — Kalb-Ramond 2-Form
Antisymmetric tensor · 28 DOF
→ Axion (solves Strong CP)

Φ — Dilaton
String coupling · $g_s = e^{\Phi}$ · 1 DOF
→ Sets all coupling constants

A_M^a — $E_8 \times E_8$ Gauge Field
496 generators · 496×8 DOF
→ All gauge bosons (g, W, Z, γ)

Ψ_M — Gravitino
M-W spinor-vector · 56 DOF
→ SUSY partner of graviton · LSP dark matter candidate

λ — Dilatino
M-W spinor · 8 DOF
→ SUSY partner of dilaton

χ^a — Gauginos
Adjoint of $E_8 \times E_8$ · 496×8 DOF
→ quarks, leptons! (upon compactification)

§2 BOSONIC SECTOR

EINSTEIN-HILBERT + DILATON

$$\mathcal{S}_{grav} = 1/2\kappa^2 \int d^{10}x \sqrt{(-g)}\, e^{-2f} (\mathcal{R} + 4\partial_m\Phi\partial^m\Phi)$$

$\mathcal{R}$ = Ricci scalar (gravity) · Φ = dilaton · e^{-2f} = string tree-level prefactor

KALB-RAMOND B-FIELD

$$\mathcal{S}_B = -1/4\kappa^2 \int d^{10}x \sqrt{(-g)}\, e^{-2f} |H_3|^2$$

$H_3 = dB_2 - \alpha'/4(\omega_3{}^{YM} - \omega_3{}^{L})$
Modified field strength — mixes gauge + gravity → anomaly cancellation

$E_8 \times E_8$ YANG-MILLS

$$\mathcal{S}_{YM} = -\alpha'/8\kappa^2 \int \sqrt{(-g)}\, e^{-2f} (\mathrm{tr}F^{(1)2} + \mathrm{tr}F^{(2)2})$$

$F^{(1)}$ = visible E_8 (→ SM forces) · $F^{(2)}$ = hidden E_8 (→ shadow matter)
248 + 248 = 496 gauge bosons · sectors couple only through gravity

§3 FERMIONIC SECTOR

GRAVITINO + DILATINO + GAUGINO KINETICS

$\bar{\psi}_m \Gamma^{MNP} \mathcal{D}_n \psi_p$ — gravitino
$+ \bar{\lambda}\Gamma^M \mathcal{D}_m \lambda$ — dilatino
$+ \alpha'/4\ \bar{\chi}^a \Gamma^M \mathcal{D}_m \chi^a$ — gaugino

KEY INTERACTION TERMS

$\frac{1}{2}\ \bar{\psi}_m \Gamma^{MNPQR} \psi_n\ H_{pQR}$ — gravitino-torsion
$\alpha'/4\ \bar{\chi}^a \Gamma^M \Gamma^{NP} \psi_m\ F_{np}$ — ← Yukawa origin!

$\chi\Gamma\psi F$ → upon compactification → all quark/lepton masses

SUSY TRANSFORMATIONS · $\mathcal{N} = 1$ · 16 SUPERCHARGES

BOSONS → FERMIONS: $\delta g_{MN} = \bar{\epsilon}\Gamma_{(M}\psi_{N)}$ $\quad \delta\Phi = \frac{1}{4}\bar{\epsilon}\lambda$ $\quad \delta A_M = \bar{\epsilon}\Gamma_M \chi$

FERMIONS → BOSONS: $\delta\psi = \mathcal{D}\epsilon - \frac{1}{8}H\Gamma\epsilon$ $\quad \delta\lambda = \Gamma\partial\Phi\epsilon$ $\quad \delta\chi = -\frac{1}{4}F\Gamma\epsilon$

§4 ANOMALY CANCELLATION — WHY $E_8 \times E_8$

THE BIANCHI IDENTITY — UNIQUELY SELECTS $E_8 \times E_8$

$$dH_3 = \alpha'/4\ (\mathrm{tr}\ \mathcal{R}\wedge\mathcal{R} - \mathrm{tr}\ F\wedge F)$$

Only $E_8\times E_8$ or SO(32) are anomaly-free in 10D

STANDARD EMBEDDING
$A_{gauge} = \omega_{spin} \Longrightarrow E_8 \to E_6 \times SU(3)$

248 DECOMPOSITION
$248 = (78,1) \oplus (27,3) \oplus (\bar{27},\bar{3}) \oplus (1,8)$

(78, 1)	E_8 gauge bosons → all forces	(27,3)+(27,3)	3 gen quarks + leptons
(1, 8)	Extra-dim curvature	$n_{gen} = \frac{1}{2}\lvert\chi(CY)\rvert$	→ topology fixes 3 gen

§5 4D EFFECTIVE THEORY

KÄHLER POTENTIAL
$\mathcal{K}(\Phi, \bar{\Phi})$
scalar kinetic terms + geometry

SUPERPOTENTIAL
$\mathcal{W}(\Phi)$
particle masses + Yukawa couplings

GAUGE KINETIC FUNCTION
$f_{ab}(\Phi)$
gauge couplings + θ-angles

THE SCALAR POTENTIAL — SOURCE OF ALL SYMMETRY BREAKING

$$V = e^{\mathcal{K}/M^2} (\mathcal{K}^{i\bar{i}} D_i\mathcal{W} D_{\bar{i}}\bar{\mathcal{W}} - 3/M^2 |\mathcal{W}|^2) + \tfrac{1}{2}(\mathrm{Re}\ f)^{-1} D^a D^b$$

F-terms → GUT breaking: $E_8 \to SO(10) \to SU(5) \to SM$ · D-terms → gauge invariance · $-3|\mathcal{W}|^2$ → cosmological constant problem

§6 COMPACTIFICATION & ROSETTA STONE

$$\mathcal{M}^{10} = \mathcal{M}^4 \times \mathcal{X}^6$$

$\mathcal{M}^4$ = our spacetime · $\mathcal{X}^6$ = Calabi-Yau (Ricci-flat, SU(3) holonomy)

TOPOLOGY → PHYSICS
$|\chi|/2$ = 3 generations · $h^{1,1}$ = moduli / inflatons · $h^{2,1}$ = Yukawa couplings · π_1 = Wilson lines → SM

ROSETTA STONE — LAGRANGIAN ⇔ SYMMETRY BREAKING POSTER

LAGRANGIAN TERM		WHAT IT PRODUCES	⇔ SYMMETRY BREAKING POSTER
$\mathrm{tr}\ F^{(1)2}$	visible Yang-Mills	78 gauge bosons → all SM forces	E6 GUT → SO(10)→SM → STANDARD MODEL
$(27,3)\oplus(\bar{27},\bar{3})$	from 248	3 families of quarks + leptons	3 GENERATIONS · UNOBS. MATTER
$V(\mathcal{K},\mathcal{W},f)$	scalar potential	drives every symmetry breaking step	SINGLET (S) · VEV MECHANISM → every ↓ arrow
$\bar{\chi}\Gamma\psi F$	Yukawa coupling	all particle masses upon compactification	QUARK / LEPTON TABLE (poster bottom)
$\mathrm{tr}\ F^{(2)2}$	hidden Yang-Mills	mirror E_8 → dark sector forces	HIDDEN SECTOR · SHADOW MATTER
$\delta\epsilon$ (SUSY)	supercharges	boson ↔ fermion pairing · LSP stability	SUSY · R-PARITY · HIGGS DOUBLETS
$(1,8) + g_{MN}$	curvature + metric	extra-dim geometry + 4D gravity	170 LEFTOVERS · 66 EXOTIC FORCES

Visualized by Prof. Andy Haas, NYU Physics, 2026